Pions in pBUU

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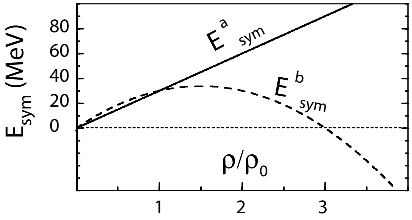
Transport 2017: International Workshop
on Transport Simulations for Heavy Ion Collisions
under Controlled Conditions

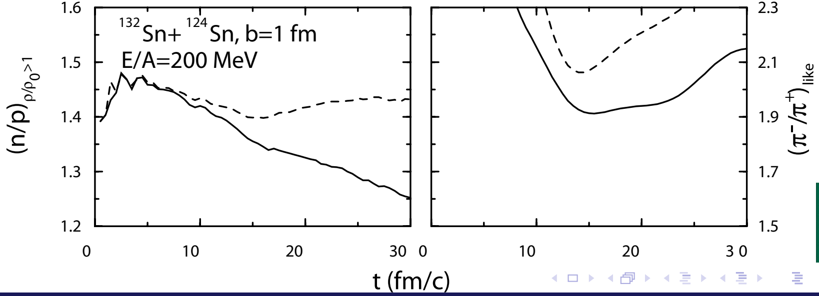
FRIB-MSU, East Lansing, Michigan, March 27 - 30, 2017

Interest: π as Probe of High-ρ Symmetry Energy

B-A Li PRL88(02)192701: $S(\rho > \rho_0) \Rightarrow n/p_{\rho>\rho_0} \Rightarrow \pi^-/\pi^+$



Pions originate from high ρ



Simulations of Heavy-Ion Collisions

Separation of time and distance scales:

Short scales reduced to negligible extent with outcomes of events treated probabilistically

Long scales treated explicitly and deterministically

Cut-off scales: $t \sim 1\,\mathrm{fm}/c$, $r \lesssim 1\,\mathrm{fm}$

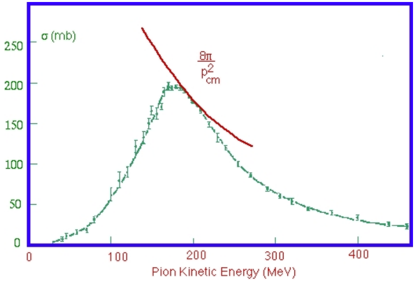
Primarily binary collision processes

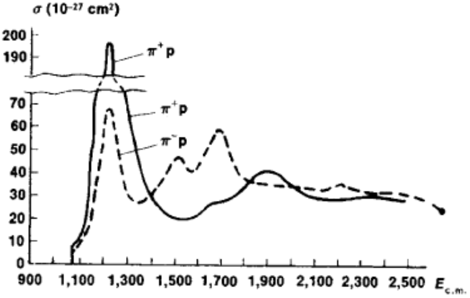
Equation of state: if there is an optical potential affecting a particle, that particle impacts the interaction parts of thermodynamic functions.

Low-E pion production: $N + N \leftrightarrow N + \Delta, \quad \Delta \longleftrightarrow N + \pi$

Δ in π–N Interactions

π–p scattering cross sections





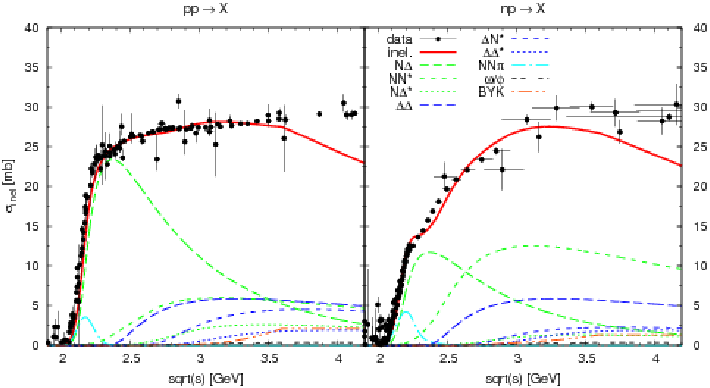
$$\sigma = \frac{\pi}{p^2}\,\frac{2J+1}{2s+1}\,\frac{\Gamma^2}{(E - m_\Delta\, c^2)^2 + \Gamma^2/4} \equiv \frac{\pi}{p^2}\,\frac{2J+1}{2s+1}\,\Gamma\,\mathcal{A}_\Delta(E)$$

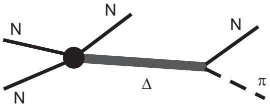
$J = 3/2$, $m_\Delta = 1232\, MeV/c^2$, $\Gamma(p) \propto p^3$, $\mathcal{A}_\Delta$ - spectral funct

Inelastic *NN* Interactions

Decomposition of inelastic *NN* cross section

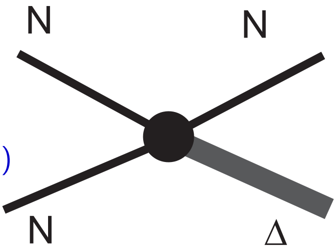
Weil *et al* EPJA48(12)111





Production and Absorption: Detailed Balance

Time reversal symmetry: same magnitude of mtx element for forward & backward process, $|\mathcal{M}_{NN\rightarrow N\Delta}| = |\mathcal{M}_{N\Delta\rightarrow NN}|$.



$$\frac{dN_\Delta}{dt} \propto \int d\boldsymbol{p}\, dm_\Delta\, \delta(\boldsymbol{p}_N + \boldsymbol{p}_\Delta - \boldsymbol{p}_N - \boldsymbol{p}_N) \times \delta(\epsilon_N + \epsilon_\Delta - \epsilon_N - \epsilon_N) \times |\mathcal{M}_{NN\rightarrow N\Delta}|^2\, (f_N\, f_N - f_N\, f_\Delta)\, \mathcal{A}_\Delta$$

in equilibrium: $f = e^{(\mu-\epsilon)/T}$

$$\sigma\, v \propto \int d\boldsymbol{p}\, dm_\Delta\, \delta(E-E)\, |\mathcal{M}|^2\, \mathcal{A}_\Delta$$

Detailed-balance relation: $\sigma_{NN\rightarrow N\Delta} \Leftrightarrow \sigma_{N\Delta\rightarrow NN}$

Relation nontrivial for Δ due to mass spread.

Balance violated: no thermal distribution, no law of mass action!

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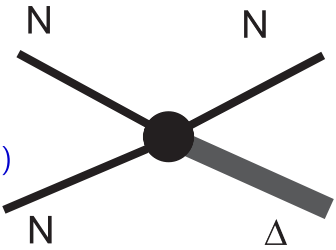

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π vs Baryon Optical Potentials

$$\Delta \longleftrightarrow N + \pi \qquad U_\Delta \stackrel{?}{=} U_N + U_\pi$$

'Conservation' of potential consistent with the quark perspective. Also also greatly facilitates calculations of process kinematics as thresholds in kinetic energy stay put.

Ferini *et al* NPA762(05)147: $U_\pi = 0$ & $U_\Delta = U_N$ employed in most models, including IBUU.

However, a strong isospin-dependent potential is needed to explain the existence of pionic atoms!

pBUU: U dependent on conserved quantities, density of baryon number and isospin - π end up with potentials that depend on isospin & symmetry energy

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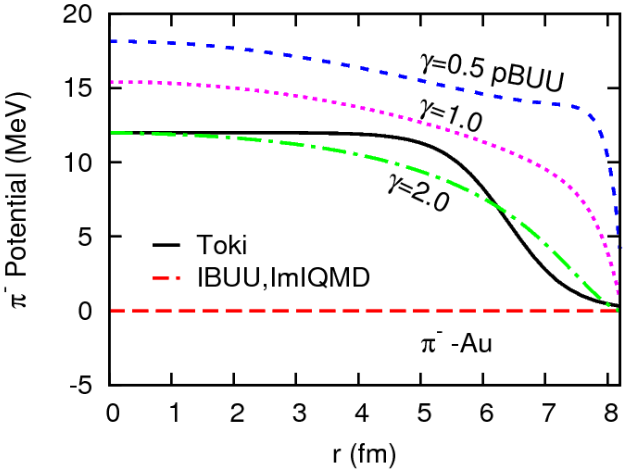
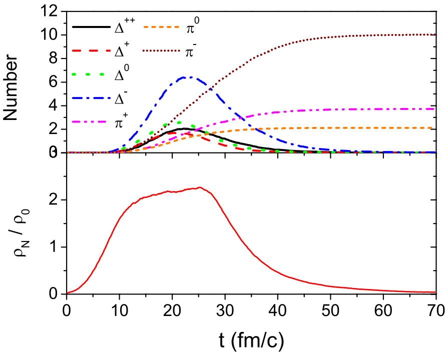
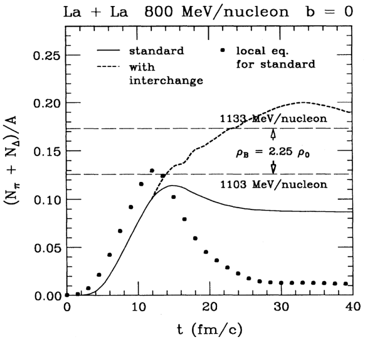
Symmetry-Energy Derived π Potential



Jun Hong&PD PRC90(14)024605 Nucl density: Thomas-Fermi

Pions Probe System at High-ρ!





Song&Ko PRC91(15)014901 PD PRC51(95)716

π test the maximal densities reached and collective motion then

Pions as Probe of High-ρ Symmetry Energy

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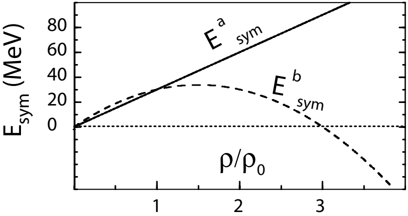

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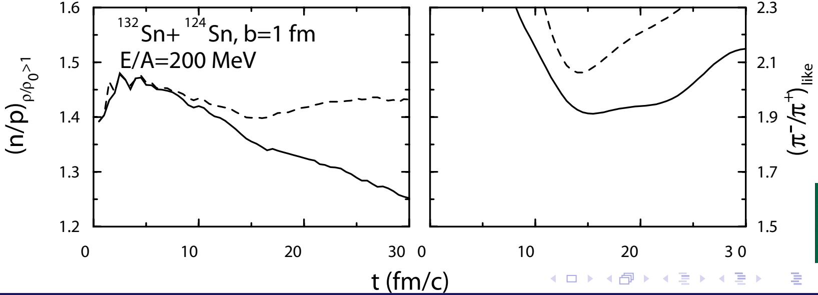
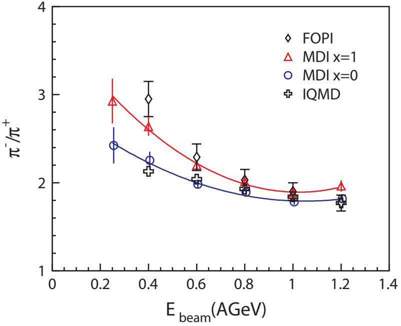
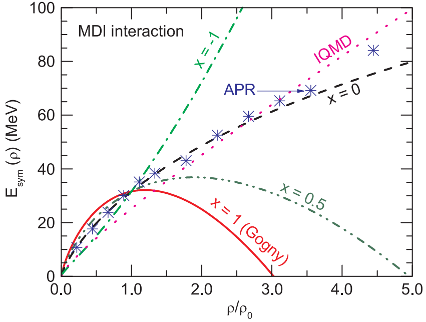


Interpretation of FOPI Data

Reisdorf *et al* NPA781(07)459



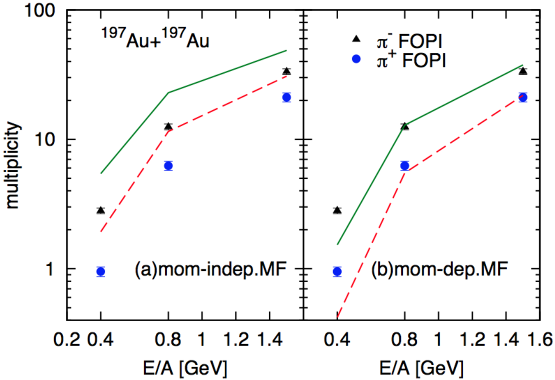


Transport IBUU04 Xiao *et al* PRL102(09)062502

Symmetry energy dropping with ρ, at $\rho > \rho_0$!?

Net π Yields and $U(\rho, p)$ in pBUU

Reisdorf *et al* NPA781(07)459

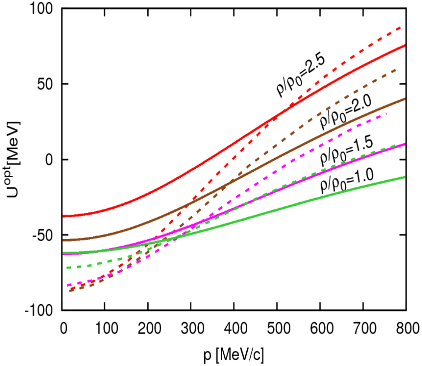


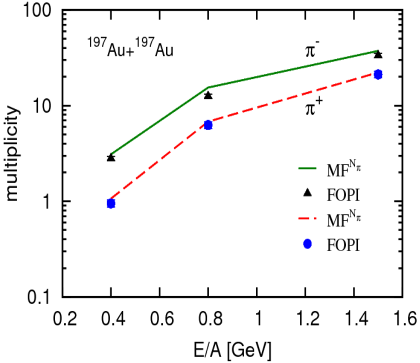
Jun Hong & PD PRC90(14)024605, π^- and π^+

?Imperfect Mom Dependence?? [No sensitivity to π/Δ rates]

affects maximal densities reached

π Yields Reproduced with Softened $U(p)$

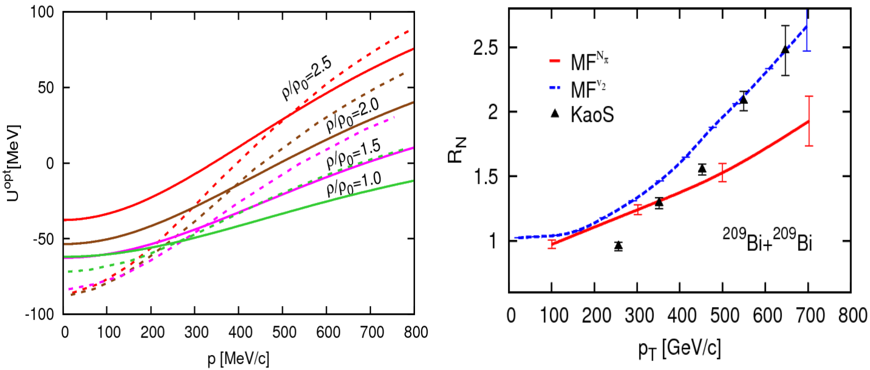




solid: softened $U(p)$ Jun Hong&PD PRC90(14)024605

but then. . .

Inferior Description of Midrapidity Flow Anisotropy



solid: new $U(p)$, dashed: old $U(p)$

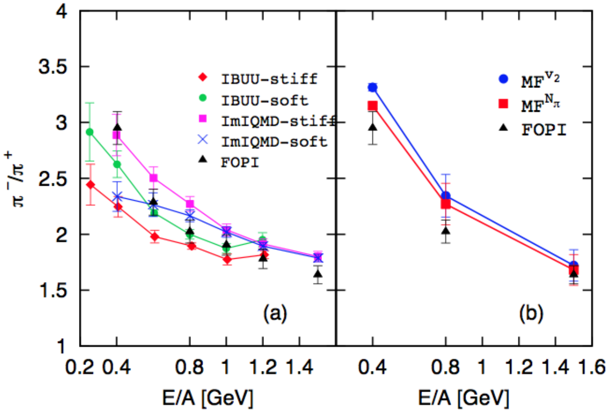
$R_N \leftrightarrow$ elliptic flow

Jun Hong & PD PRC90(14)024605

too weak with new $U(p)$

FOPI π^-/π^+ Reproduced by pBUU

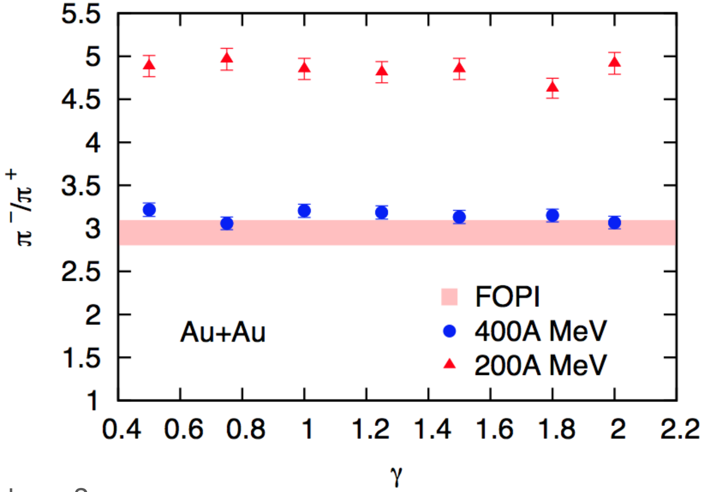
...irrespectively of $U(p)$, right panel



Left panel: discrepancies in the literature - correlation vs anticorrelation of $S(\rho > \rho_0)$ with π^-/π^+.

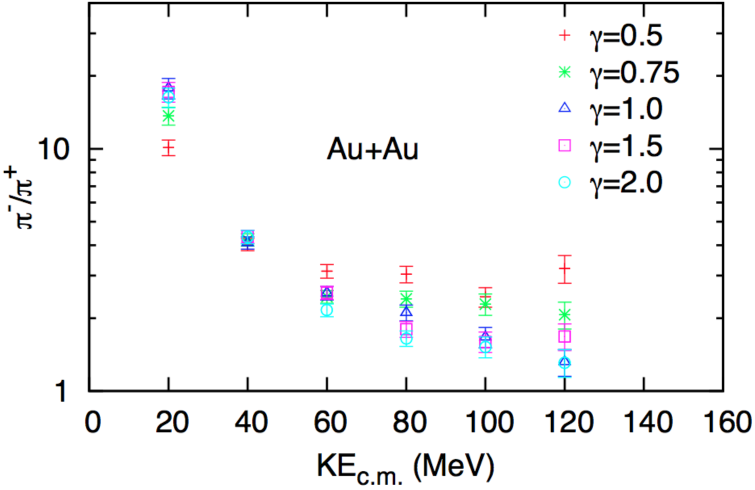
FOPI π^-/π^+ Reproduced by pBUU

...irrespectively of $S_{\rm int}(\rho) = S_0\,(\rho/\rho_0)^\gamma$:



?no hope?

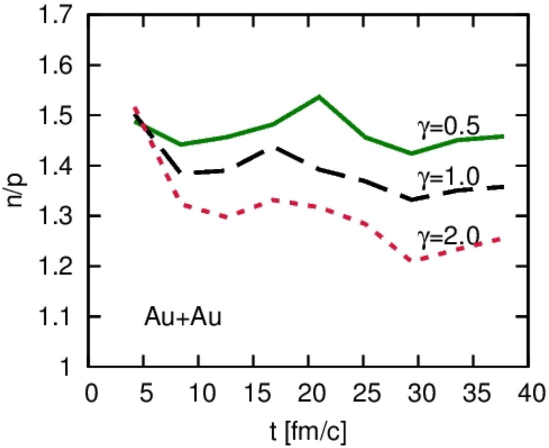
Original Idea Still Correct for High-E π's



$S_{\text{int}}(\rho) = S_0\,(\rho/\rho_0)^\gamma$

n/p Ratio in pBUU at $\rho > \rho_0$

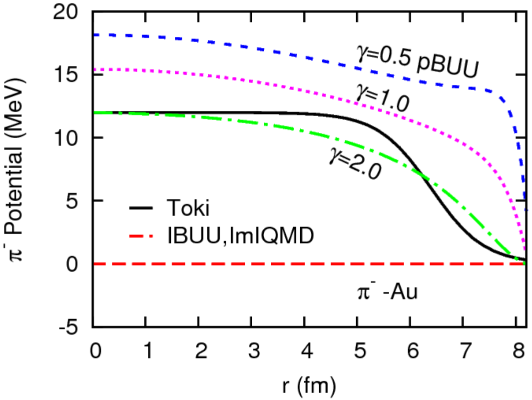
changes with the supranormal symmetry energy:



$S_{\text{int}}(\rho) = S_0\,(\rho/\rho_0)^\gamma$

Why Differences for Net π Ratios?

In pBUU isospin-driven $\pi^{\pm}$ optical potential

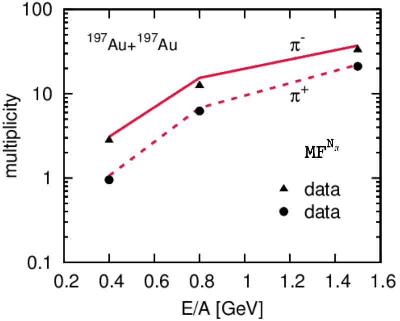


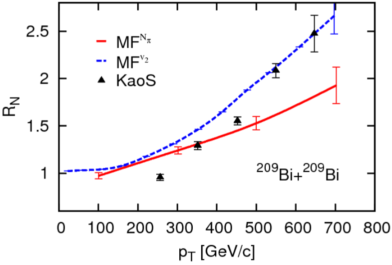
π/Δ rate sensitivities claimed in Larionov&Mosel NPA728(03)135; Prassa *et al* NPA789(07)311 and Song&Ko PRC91(15)014901. Virtually none there in pBUU!

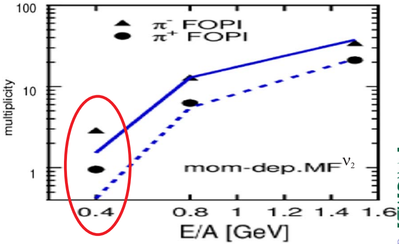
Paradox: Elliptic Flow vs π Yields

Changing mo-dep of MF:
either v_2 good
or near-threshold M_π,
but not both!

$$R_N = \frac{1 - v_2}{1 + v_2}$$



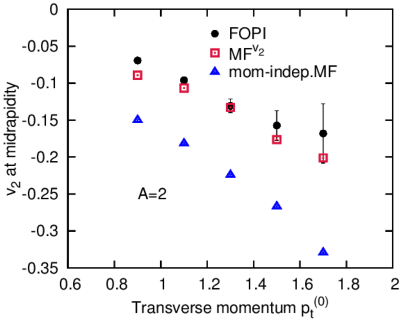




Tinkering with Incompressibility

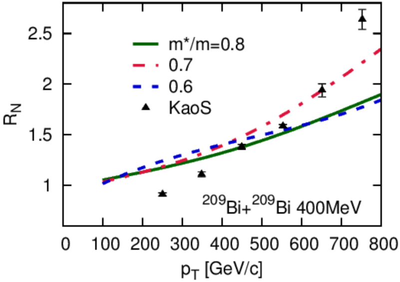
Results so far
for $K = 210\,\text{MeV}$.

While elliptic flow is more sensitive to the momentum dependence of mean field, or m^*/m, the sensitivity to incompressibility K is also there!

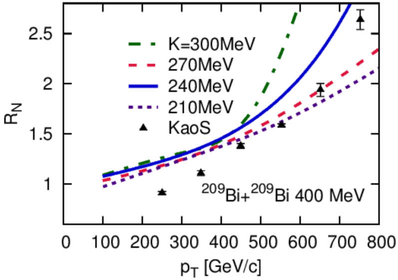


$K = 380\,\text{MeV}$

Sensitivity of Elliptic Flow to m^*/m and K



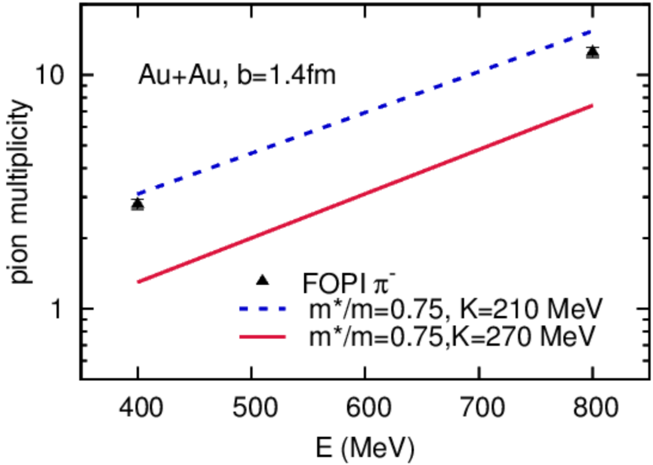
$K = 270\,\mathrm{MeV}$
and changing m^*/m



$m^*/m = 0.7$
and changing K

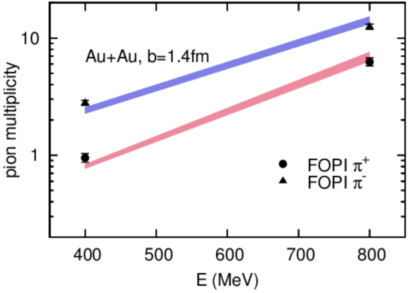
Hysteresis in both cases due to competition between density and momentum dependence

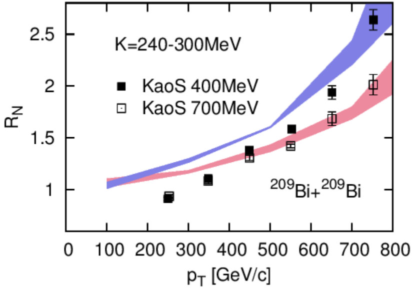
Sensitivity of M_π to Incompressibility K



$m^*/m = 0.75$ and changing K

Raising K Allows to Describe Both M_π and v_2!

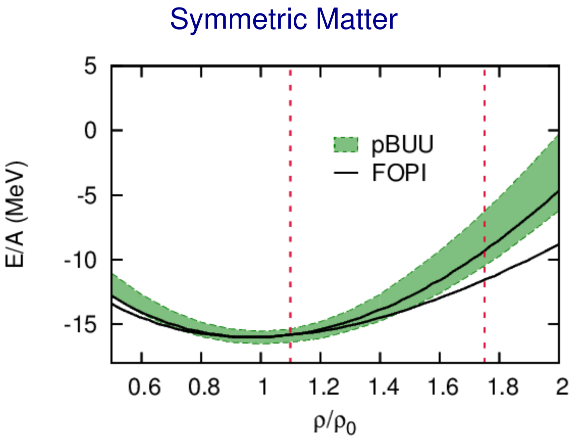




Bands for $K = (240 - 300)$ MeV & optimal m^*/m

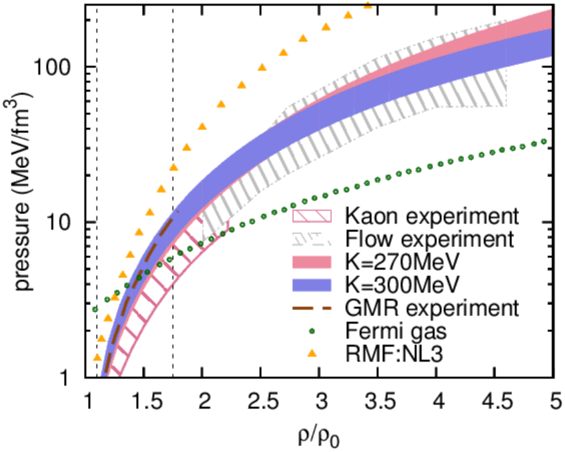
$\rightarrow$ Constraints on EOS, at moderately supranormal densities, à la LeFèvre *et al*

Energy Per Nucleon



Pressure

Symmetric Matter



Conclusions

- Detailed balance must be obeyed for thermodynamic consistency
- Uncertainties in the near-threshold π production include π & Δ optical potentials & in-medium rates
- Pions probe high-ρ matter, net density, n/p-ratio, collective flow there! . . . $U(p)$ & K
- pBUU reproduces FOPI π^-/π^+, irrespectively of details in U and S
- High-energy π^+/π^- ratio more robust than ratio of net yields
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